

B.Sc.(Srimester-3)
Subject: Physics
Course: US03CPHY01
Title: Optics

UNIT – III Polarization

❖ **Introduction:-**

- Interference and diffraction phenomena proved that light is a wave motion. These phenomena are used to find wavelength of light. However, they do not give any indication regarding the character of waves.
- Maxwell developed electromagnetic theory and suggested that light-waves are electromagnetic waves. Electromagnetic waves are transverse waves, so it is obvious that light waves are also transverse waves.
- Longitudinal waves are waves in which particles of medium oscillate along the direction of propagation of wave (e.g. sound wave).
- Transverse waves are waves in which particles of medium oscillate perpendicular to the direction of propagation of wave. (e.g. Electromagnetic waves.)
- Polarization is possible in transverse wave.
- **Unpolarized Light** is the light in which the planes of vibration are symmetrically distributed about the propagation direction of the wave.
- **Plane Polarized light** is a wave in which the electric vector is everywhere confined to a single plane.
- **A linearly polarized light** wave is a wave in which the electric vector oscillates in a given constant orientation.

❖ **Production of Linearly Polarized Light:**

Linearly polarized light may be produced from unpolarized light using following optical phenomena.

- (i) Reflection
- (ii) Refraction
- (iii) Scattering
- (iv) Selective absorption
- (v) Double reflection.

Polarized light has many important applications in industry and engineering. One of the most important applications is in liquid crystal displays (LCDs) which are widely used in wristwatches, calculators, T.V. Screens etc. An understanding of polarization is essential for understanding the propagation of electromagnetic waves guided through wave-guides and optical fibers.

➤ **Polarization by reflection:-**

In 1808, E.L.Malus discovered the polarization of natural light by reflection from the surface of glass. He noticed that when natural light is incident on a plane sheet of glass at a certain angle, the reflected beam is plane polarized.

Figure 1 shows an unpolarised light beam AB incident on a glass surface. The incident ray AB and the normal NBN' represent the plane of incidence. The incident

ray AB resolved into two components. (i) The perpendicular component, and (ii) parallel component. The perpendicular component is represented by dots and called the **s-component**. The parallel component is represented by the arrows and is called the **p-component**.

The reflected ray BC has predominance of s-component and it is partially polarized as shown in fig(a). As the angle of incidence is varied, the polarization is also varied. At a particular angle θ_p , the reflection coefficient for p-component goes to zero and the reflected beam does not contain any p-component. It contains only s-component and is totally *linearly polarized*. The angle θ_p is called the **polarizing angle**.

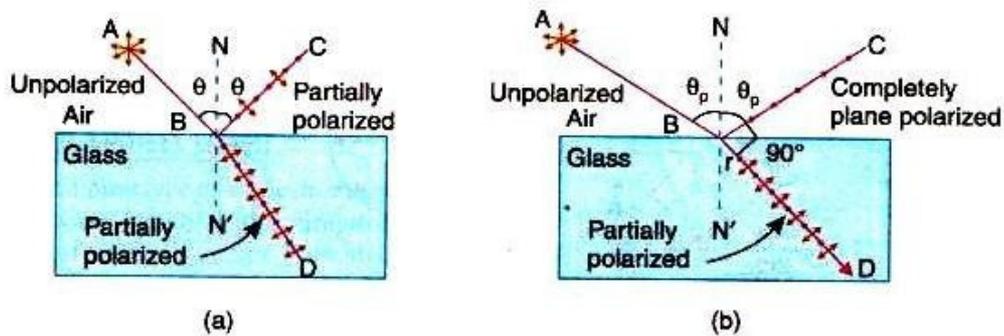


Figure:1

In the path of 'BC', place a tourmaline crystal and rotate it slowly. It will be observed that light is completely extinguished only at one particular angle of incidence. This angle of incidence is known as the polarizing angle.

❖ Brewster's Law:-

In 1892, Brewster performed number of experiments to study the polarization of light by reflection at the surfaces of different media.

He found that ordinary light is completely polarized in the plane of incidence when it gets reflected from a transparent medium at a particular angle known as the '**angle of polarization**.'

He proved that '**the tangent of the angle of polarization is numerically equal to the refractive index of the medium.**' Also the reflected and refracted rays are perpendicular to each other.

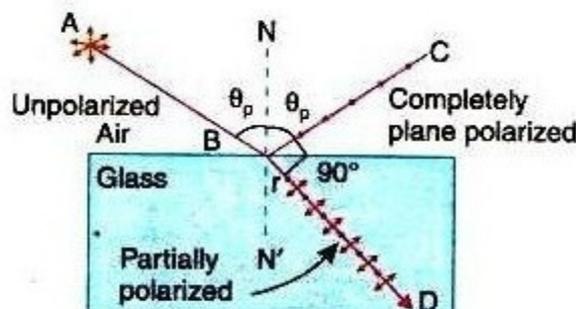


Figure:2

If θ_p is the angle and μ is the refractive index of the medium, then

$$\mu = \tan\theta_p$$

This is known as Brewster's law.

If natural light is incident on smooth surface at the polarizing angle, it is reflected along BC and refracted along BD as shown in figure 2. Brewster found that the maximum polarization of reflected ray occurs when it is at right angles to the refracted ray. It means that $\theta_p + r = 90^\circ$

According to Snell's law,

$$\frac{\sin \theta_p}{\sin r} = \frac{\mu_2}{\mu_1}$$

Where, μ_2 is the absolute refractive index of reflecting surface and μ_1 is the refractive index of the surrounding medium.

Hence,

$$\frac{\sin \theta_p}{\sin(90^\circ - \theta_p)} = \frac{\mu_2}{\mu_1}$$

$$\frac{\sin \theta_p}{\cos \theta_p} = \frac{\mu_2}{\mu_1}$$

$$\tan \theta_p = \frac{\mu_2}{\mu_1}$$

Above equation shows that the polarizing angle depends on the refractive index of the reflecting surface. The polarizing angle θ_p is also known as **Brewster angle** and denoted by θ_B . The light reflected from any angle other than Brewster angle is partially polarized.

As shown in the figure, unpolarized light is incident at our angle equal to the polarizing angle on the glass surface. It is reflected along 'BC' and refracted along 'BD'.

From Snell's law,

$$\mu = \frac{\sin i}{\sin r} \quad (1)$$

From Brewster's law,

$$\mu = \tan i = \frac{\sin i}{\cos i} \quad (2)$$

Comparing (1) & (2),

$$\cos i = \sin r = \cos\left(\frac{\pi}{2} - r\right)$$

$$\therefore i = \frac{\pi}{2} - r$$

$$\therefore i + r = \frac{\pi}{2}$$

As $i + r = \frac{\pi}{2}$ angle CBD is also equal to $\frac{\pi}{2}$.

Therefore the reflected and the refracted rays are at right angles to each other.

From Brewster's law, the value of 'i' for crown glass of refractive index 1.52 is given by.

$$i = \tan^{-1}(1.52)$$

$$\therefore i = 56.7^\circ$$

For ordinary glass the approximate value for the polarizing angle is 57° . For a refractive index of 1.7, the polarizing angle is 59.5° . Thus the polarizing angle is not widely different for different glasses.

As the refractive index of a substance varies with the wavelength of the incident light, the polarizing angle will be different for light of different wave length.

➤ Applications of Brewster's law:-

1. Brewster's law can be used to determine the reflective Indices of opaque materials.
2. It is used to calculate the polarizing angle for total polarization of reflected light, if reflective index of the material is known.
3. Brewster's angle can be utilized for transmitting a light beam in into or out of an optical fiber without reflections losses.

❖ Polarization by refraction-pile of plates

When unpolarized light is incident at Brewster's angle on a smooth glass surface, the reflected light is totally polarized while the refracted light is partially polarized. If natural light is transmitted through a single plate, they it is partially polarized. If a stack of glass plates is used instead of a single plates is used instead of a single plate, reflections from successive surfaces of each glass plate filter the perpendicular component from the transmitted ray. So, transmitted ray consists of only parallel components.

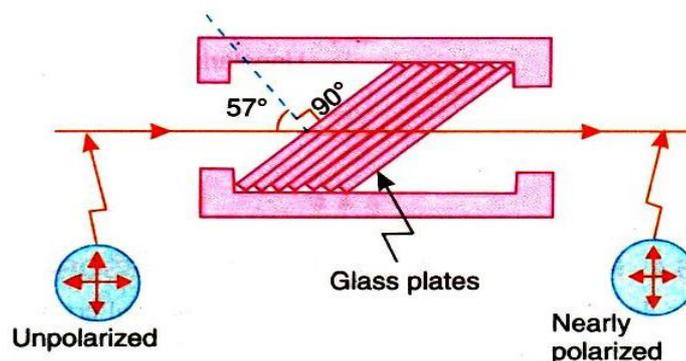


Figure:3

I_p - Intensity of parallel component of refracted light.

I_s - intensity of perpendicular component of light.

Thus, degree of polarization of refracted (transmitted) light is given by

$$P = \frac{I_p - I_s}{I_p + I_s} = \frac{m}{m + \left(\frac{2\mu}{1 - \mu^2}\right)^2}$$

where m = no. of plates required.

μ = refractive index of material.

About 15 glass plates are required for this purpose. Thus glass plates are kept inclined at an angle of 33° to the axis of the tube as shown in figure 3. Such an arrangement is called a **pile of plates**. When unpolarized right is incident on the plates at Brewster angle, the transmitted light will be polarized and parallel to the plane of incidence.

The drawback of this method is good portion of light is lost in reflections.

❖ Polarization by Scattering

If a narrow beam of natural light is incident on a transparent medium, contain a suspension of ultramicroscopic particles, the scattered light is partially polarized. The degree of polarization depends on the angle of scattering. The beam scattered at 90° with respect to the incident direction is linearly polarized.

The direction of vibration of E vector in the scattered light will be perpendicular to the plane of propagation of light. Sun light scattered by air molecules is polarized. The maximum effect is observed on a clear day when the sun is near the horizon. The light reaching on the ground from directly overhead is polarized to the extent of 70% to 80%.

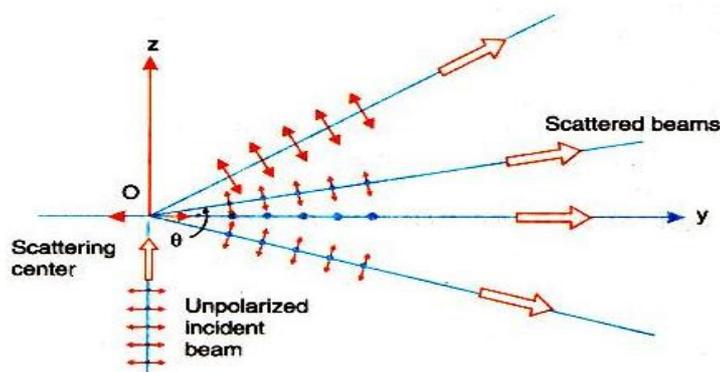


Figure: 4

❖ Polarization by Selective absorption

In 1815 Biot discovered that certain mineral crystal absorbs light selectively. When natural light passes through a crystal such as tourmaline, it splits into two components which are polarized in mutually perpendicular places.

The crystal strongly absorbs light that is polarized in a direction parallel to a particular plane in the crystal but freely transmits the light component polarized in a perpendicular direction. This difference in the absorption for the rays is known as **selective absorption or dichroism**.

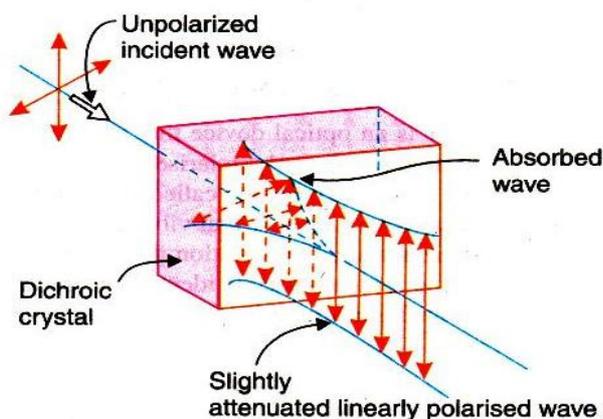


Figure: 5

If the crystal is of proper thickness, one of the components is totally absorbed and the other component emerging from the crystal is linearly polarized. It is shown in the figure 5.

The difference in absorption in different direction may be understood from the electron theory. When the frequency of incident light wave is close to natural frequency of the electron cloud, the light waves are absorbed strongly. Crystals that exhibit selective absorption are **anisotropic**.

The crystal splits the incident wave into two waves. The component having its vibration perpendicular to the principal plane of the crystal gets absorbed. The component with parallel vibrations is less absorbed and it is transmitted. The transmitted light is linearly polarized. The drawback of this method is that the crystal of bigger size cannot be grown.

❖ Polarization by double refraction

This phenomenon was discovered by **Erasmus Bartholinus** in 1669. When light is incident on a calcite crystal, it splits into two refracted rays. This phenomenon is called **double refraction** or **birefringence**. The crystal is called **birefringent**.

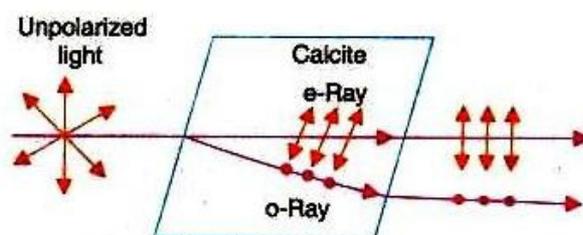


Figure: 6

The two rays produced in double refraction are linearly polarized in mutually perpendicular directions. The ray which obeys Snell's law of refraction is known as **ordinary ray** or **o-ray**. The other ray does not obey Snell's law is called **extraordinary ray** or **e-ray**.

❖ Polarizer and Analyzer :-

➤ Polarizer:-

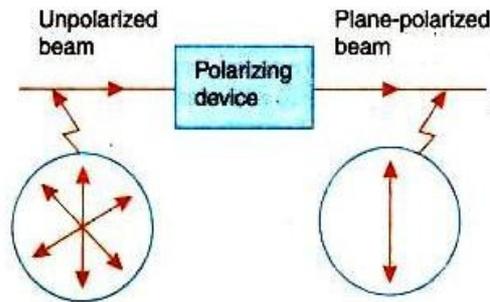
It is an optical device that transforms unpolarized light into polarized light. If it produces linearly polarized light. It is called a linearly Polarizer.

If natural light is incident on a linear polarizer, only that vibration which is parallel to the transmission axis is allowed to pass through the polarizer while the vibration that is in a perpendicular direction is totally blocked.

➤ Analyzer:-

Analyzer is a device, which is used to find whether the light is polarized or unpolarized.

Both polarizer and analyzer are fabricated in the same way and have the same effect on the incident light.

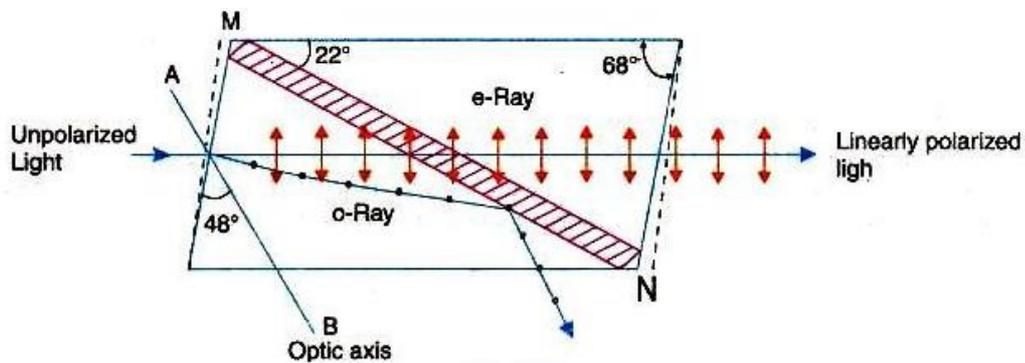


A polarizer transforms unpolarized light into polarized light.

➤ NICOL'S PRISM:-

A Nicol prism is made from calcite crystal. It was designed by **William Nicol** in 1820.

A calcite crystal whose length is three times as its width is taken. The end faces of this crystal are ground in such a way that the angles in the principal section become 68° and 112° instead of 71° and 109° the crystal is cut in two pieces by a plane perpendicular to the principal section as well as the new end faces. The two parts of the crystal are then cemented together with Canada balsam. The refractive index of Canada balsam lies between the refractive indices for the ordinary and extraordinary rays for calcite.



$$(\mu_o=1.66, \mu_e=1.486 \text{ and } \mu_{\text{Canada balsam}} = 1.55)$$

The position of optic axis AB is as shown in figure.

Unpolarized light is made to fall on the crystal as shown in figure. The ray after entering the crystal suffers double refraction and splits up into O-ray and e-ray. The values of the refractive indices and the angles of incidence at the Canada balsam layer are such that the e-ray is transmitted while the o-ray is internally reflected. The face where the o-ray is incident is black so that the o-ray is completely absorbed. Thus we get only the linearly polarized e-ray coming out of the Nicol prism with the direction of vibration as shown in figure. Thus, the Nicol Prism works as a polarizer.

The Nicol prism is the most widely used polarizing device. It is a good polarizer but expensive. It has a limited field of view of about 28° .

➤ Polaroid Sheets:-

In 1928 **E.H. Laud** invented a method of aligning small crystal to obtain large polarizing sheets. The sheets are called Polaroid sheets.

Constructions:-

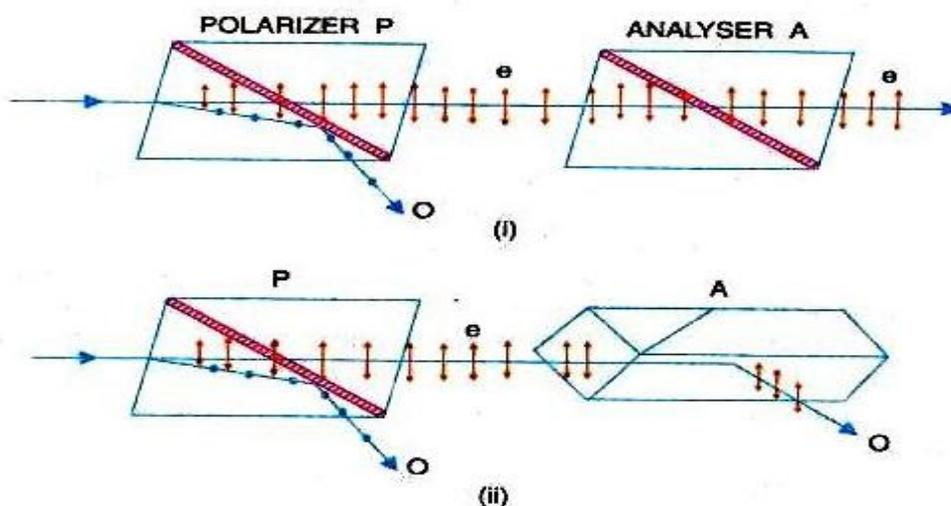
- A clear plastic sheet of long chain molecules of polyvinyl alcohol (PVA) is heated.
- It is then stretched in a given direction to many times its original length.
- During this process, PVA molecules become aligned along the direction of stretching.
- The sheet is there laminated to a rigid sheet of plastic.
- It is then exposed to iodine vapour.
- The iodine atoms attach themselves to the straight long chain of PVA molecules.
- The conduction electrons associated with iodine can move along the chains.
- A sheet fabricated according to this process is known as H-sheet.

➤ Working of Nicol Prism as Polarizer and Analyzer:-

Nicol Prism was invented by **William Nicol** in 1828. It is an optical device used for producing and analyzing plane polarized light. When a beam of light passes through a calcite crystal, it breaks up into o-ray and E-ray.

The Nicol prism is designed in such a way that it eliminates O-ray. Hence E-ray are transmitted through the prism.

It is used for the production and detection of plane polarized light.



In above figure two Nicol Prisms are shown placed near by to each other 'P', works as polarizer and it produces plane polarized light 'A' works as an analyzer and it analyses whether the emerging rays from 'P', is plane polarized, circularly polarized or elliptically polarized.

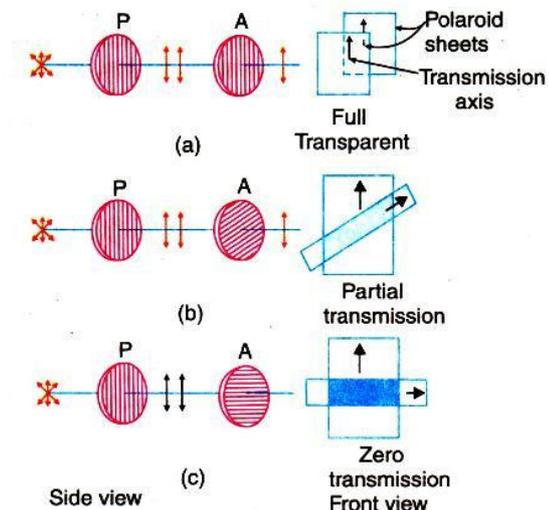
As shown in fig-(i) o-ray is reflected by 'P₁' would E-ray is transmitted through it. As shown in fig.-(ii) if 'P₂' is gradually rotated the intensity of E-ray decreases and when the two prisms are crossed, then no light comes out of 'P₂'.

It means that light coming out of 'P₁' is plane polarized. When the polarized E-ray enters 'P₂' it acts as an o-ray and it is then totally internally reflected by the Canada balsam layer. Hence light will not come out of 'P₁'. Therefore the prism 'P₁' produces plane polarized light and prism 'P₂' detect (analyze) it.

➤ Polaroid Sheets as Polarizer and Analyzer:

Fig. shows the Polaroid sheets in three different configurations.

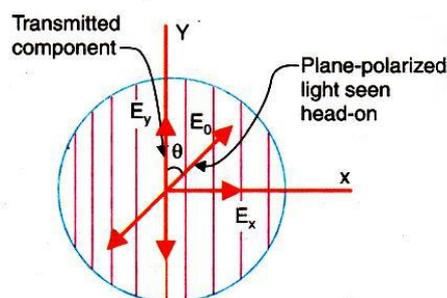
- Fig (a) - transmission axis of the analyzer. A is parallel to polarizer P so light passes through the analyzer.
- Fig (b) Transmission axis is at angle θ so light partially transmitted.
- Fig(c) When the axes are perpendicular to each other, no light is transmitted.



➤ Working

- When natural light is incident on the sheet, the component that is parallel to the chains of iodine atoms induces current in the conducting chains and is therefore strongly absorbed.
- Thus the transmitted light contains only the component that is perpendicular to the direction of molecular chains.
- The direction of E-vector in the transmitted beam corresponds to the transmission axis of the Polaroid sheet.
- These sheets are expensive and can be made in large sizes.
- They are widely used in sunglasses, camera filters etc. to eliminate the unwanted glare from objects.
- They can be used as polarizer and analyzer.
- Effect of Polarizer on natural light.

When unpolarized light passes through a polarizer, the intensity of the transmitted light will be exactly half that of the incident light. This can be proved as follows.



Let E_0 is the amplitude of vibration of one of the waves of the unpolarized light incident on the polarizer.

Let E_0 makes an angle Q with the transmission axis of the polarizer.

Here E_0 may be resolved into its rectangular components E_x and E_y as shown in fig.

The polarizer blocks the component E_x and transmits the component E_y .

The intensity of the transmitted light is then,

$$I = E_y^2 (\cos \theta)^2$$

$$= E_0^2 (\cos^2 \theta)$$

(Only E_y component of E_0 is transmitted)

But,

$$\text{Intensity} \propto (\text{Amplitude})^2$$

Therefore,

$$I = I_0^2 (\cos^2 \theta)$$

Now, the value Q is from 0 to 2π because unpolarized light has all possible vibrations.

$$I = I_0^2 \langle \cos^2 \theta \rangle$$

$$= \frac{I_0}{2\pi} \times \int_0^{2\pi} \cos^2 \theta \, d\theta$$

$$= \frac{I_0}{2\pi} \times \int_0^{2\pi} \frac{1 + \cos 2\theta}{2} \, d\theta$$

$$I = \frac{I_0}{4\pi} \left[\int_0^{2\pi} d\theta + \int_0^{2\pi} \frac{1 + \cos 2\theta}{2} \, d\theta \right]$$

$$I = \frac{I_0}{4\pi} \left([\theta]_0^{2\pi} + \left[\frac{\sin 2\theta}{2} \right]_0^{2\pi} \right)$$

$$I = \frac{I_0}{4\pi} (2\pi + 0)$$

$$I = \frac{I_0}{2}$$

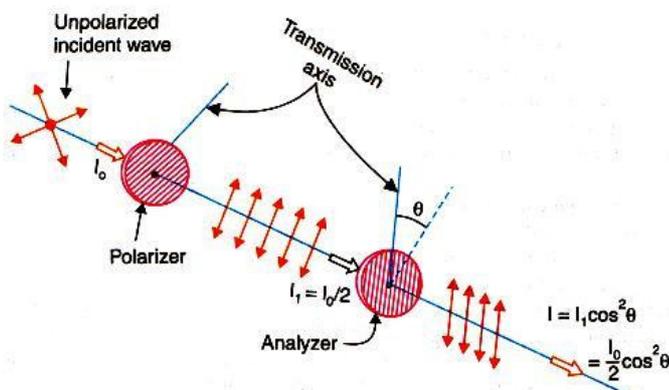
Thus, the intensity of transmitted light through the polarizer is half the intensity of incident light.

➤ Effect of analyzer on plane polarized light- Malus law

When natural light is incident on a polarizer, the transmitted light is linearly polarized. If this light-further passes through an analyzer, the intensity varies with the angle between the transmissions axes of polarizer, and analyzer. Malus studied their phenomenon and known as Malus law.

It states that

"The intensity of the polarized light transmitted through the analyzer is proportional to cosine square of the angle between the plane of transmission of the analyzer and plane of transmission of the polarizer."



Let I_0 is the intensity of unpolarized light. The intensity of polarized light from the polarizer is $I_0/2$. Take $I_1 = I_0/2$.

This plane polarized light then passes through the analyzer. Let E is the aptitude of vibration and Q is the angle between this vibration and transmission axis of an analyzer. E resolves into two components (i) E_y , parallel to the plane of transmission of the plane of analyzer and (ii) E_x , perpendicular to the plane of analyzer.

Now, E_y component is only transmitted through the analyzer.

$$E_y = E \cos \theta$$

Intensity of light for this component:

$$I = E^2 \cos^2 \theta = I_1 \cos^2 \theta = \left(\frac{I_0}{2}\right) \cos^2 \theta$$

If,

- (i) $\theta = 0^\circ$, then axis are parallel $I = I_1$
- (ii) $\theta = 90^\circ$, then axis are perpendicular $I = 0$
- (iii) $\theta = 180^\circ$, then axis are parallel $I = I_1$
- (iv) $\theta = 270^\circ$, then axis are perpendicular $I = 0$

Thus, there are two positions of maximum intensity and two positions of zero intensity when we rotate the axis of the analyzer with respect to that of the polarizer.

❖ Anisotropic Crystals:

- **Isotropic Materials:-**

In isotropic materials atoms are arranged in a regular periodic manner. In isotropic materials, when a light beam is incident, it refracts a single ray. It means that in such material the refractive index is same in all direction. e. g. Glass water and air

- **Anisotropic Materials:-**

In anisotropic material the arrangement of atoms differs in different directions within a crystal. Thus the physical properties vary like, thermal

conductivity, electrical conductivity, velocity of light and have refractive index etc. vary with the directions. Such crystal are then said Anisotropic.

The anisotropic crystals are divided into two classes.

(i) Uniaxial Crystal:

In this type of crystal, one of the refracted rays is an ordinary ray and the other is an extraordinary. e. g. Calcite, tourmaline and Quartz.

(ii) Biaxial Crystal:

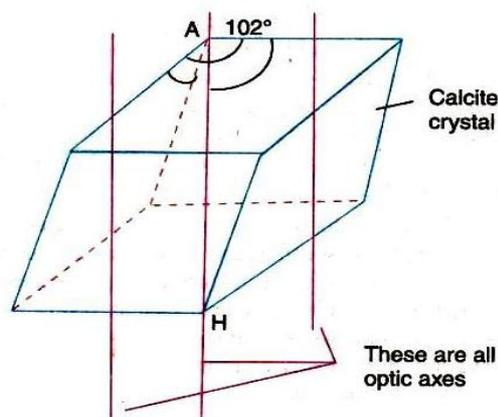
In biaxial crystal both the refracted rays are extra ordinary rays. e. g. mica, topaz & aragonite.

➤ Calcite Crystal:

Figure shows a calcite crystal in the form of a rhombohedron bounded by six parallelograms with angles equal to 78° and 102° . At two opposite corners (A&H) the three angles of faces meeting there are all obtuse. These corners (A&H) are known as blunt corners.

• Optic axis

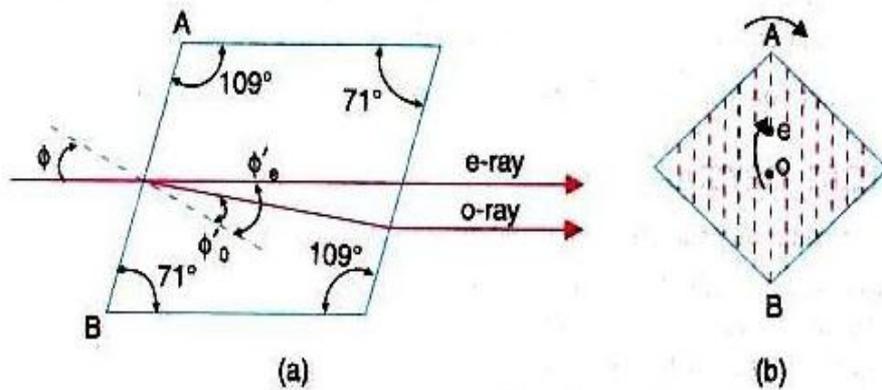
A line passing through 'A' making equal angles with each of the three corners gives the direction of optic axis. Any line parallel to this line is also an optic axis. In the figure AH is the optic axis of calcite crystal. If a ray of light is incident along the optic axis or in a direction parallel to the optic axis, then it will not split up into two rays. Thus the phenomenon of double refraction is absent when the light is allowed to enter the crystal along the optic axis.



➤ Principal Section

A plane containing the optic axis and perpendicular to a pair of opposite faces of the crystal is called the principal section of the crystal for that pair of faces.

As a crystal has six faces, so far every point inside the crystal there are three principal sections, one for each pair of opposite crystal faces. A principal section cuts the crystal surfaces in a parallelogram having angles 71° and 109° .



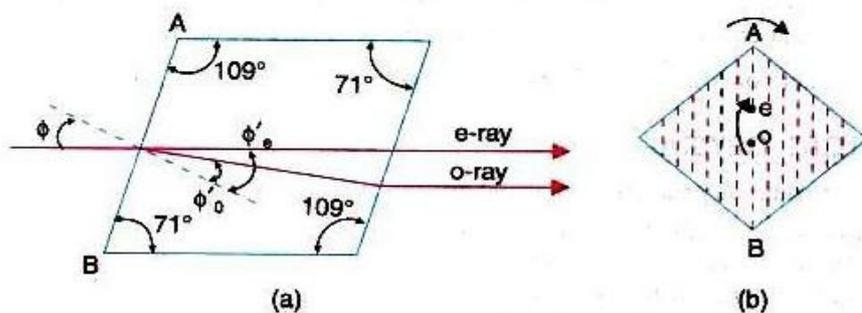
In fig(a) the principal section of the crystal is shown. An end view of any principal section is a straight line (shown by dotted line in fig.b)

The plane containing the optical axis and the O -ray is called the principal plane of O -ray. The plane containing the optic axis and the E-ray is called the principal plane of E-ray.

➤ Double refraction:-

When a ray of light is refracted by a crystal of calcite it gives two refracted rays. This phenomenon is called "**DOUBLE REFRACTION**".

The phenomenon of double refraction can be shown with the help of the following experiment.



Positive crystal: When refractive index for extraordinary ray is greater than that of O-ray $\mu_e > \mu_o$.

Negative crystal: when refractive index for extraordinary ray is lesser than that of O-ray $\mu_e < \mu_o$

Mark an ink dot on a piece of paper. If we place a calcite crystal over this dot, then two images of dots are observed. Now rotate the crystal slowly as shown in figure ii. It is found that our image remains stationary ink the second image rotates with the rotation of the crystal. The stationary image is known as the ordinary image while the second image is known as the extraordinary image. The refracted ray which produces ordinary image is known as **ordinary ray O-ray** and the refracted ray which produces extraordinary image is known **extraordinary ray (E-ray)**.

When a ray of light AB is incident in the calcite crystal making our angle of incidence i , it is refracted along two paths inside the crystal

- (i) Along BC making our angle of refraction r_2 and

(ii) Along BD making our angle of refraction r_1 .
These two rays emerge out along CE and DO are parallel.

The difference between o-ray and e-ray is given below:

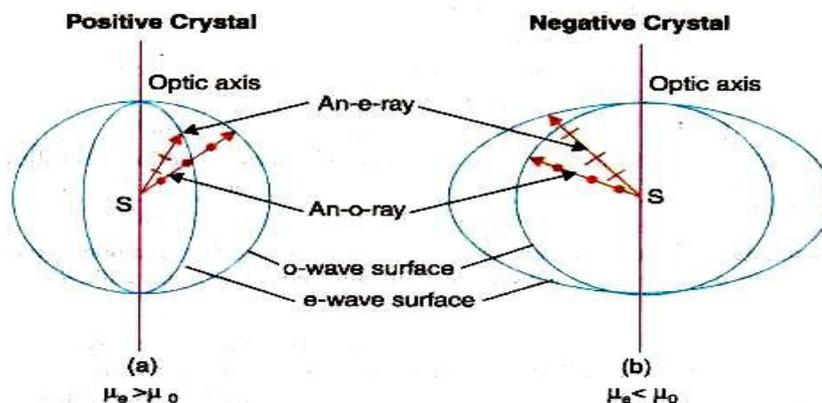
- 1) The ordinary ray has a refractive index $\mu_o = \frac{\sin i}{\sin r_1}$ and the extraordinary ray has a refractive index $\mu_e = \frac{\sin i}{\sin r_2}$.
- 2) The O-ray obeys the laws of refraction and its refractive index is constant. For E-ray its refractive index varies with the angle of incidence and it is not fixed.
- 3) For the case of calcite $\mu_o > \mu_e$ because r_1 less than r_2 . Therefore, the velocity of light for the O-ray inside the crystal is less than the velocity of light for E-ray. ($\mu_o = \frac{c}{v_o}$ and $\mu_e = \frac{c}{v_e}$).
- 4) The O-ray travels in the crystal with same velocity in all directions; where as the velocity of E-ray is different in different directions, because its refractive index varies.
- 5) Both O-ray and E-ray is plane with the angle of incidence. Plane polarized they are polarized in mutually perpendicular planes.

❖ Huygen's explanation of double refraction in uniaxial crystal

According to Huygen's they each point on a wave front acts as a fresh source of disturbance and said secondary wavelets. He explained the phenomena of double refraction in Uniaxial crystal with the help of secondary wavelets.

According to his theory:

- 1) When any wave front strikes a doubly refraction, crystal every point of the crystal becomes a course of two wave fronts.
- 2) Ordinary wave front corresponding to ordinary rays. Since ordinary rays have same velocity in all directions, the secondary wave front is spherical.
- 3) Extra-ordinary wave front corresponding to extra-ordinary rays. Since extra-ordinary rays have different velocities in different directions, the extra-ordinary wave front is ellipsoid of revolution, with optic axis as the axis of revolution.
- 4) The sphere and ellipsoid touch each other at points which lie on the optic axis of the crystal, because two velocity of ordinary and extra ordinary ray is same along the optic axis.



Huygens wave surfaces produced by a point source S -embedded in the birefringent crystal; (a) a positive crystal (b) a negative crystal.

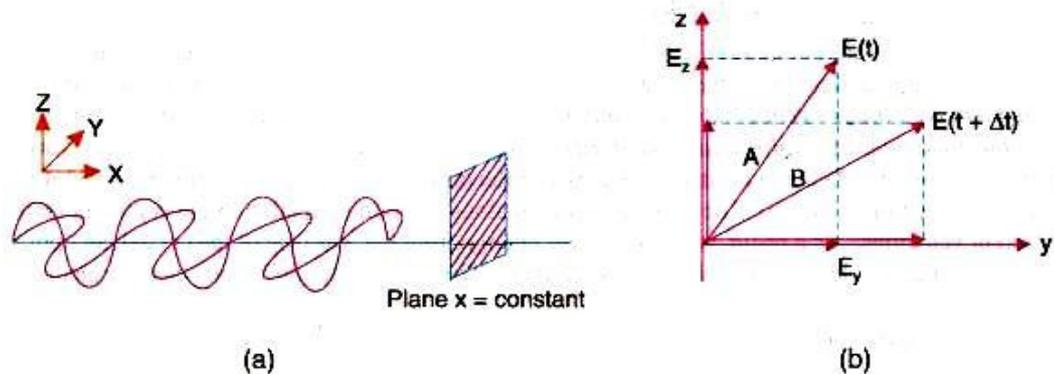
- 5) In certain crystals like calcite and tourmaline called the negative crystal, the ellipsoid lies outside the sphere as shown in fig.(a).
- 6) This shows that in negative crystals, the extra-ordinary wave front travels faster than ordinary wave front except along optic axis.
- 7) ($V_o > V_e$ and $\mu_o > \mu_e$).
- 8) in certain crystal (like quarter). Sphere lies outside the ellipsoid as shown in fig-b. Such crystals are called positive crystals. In the crystals, variety of ordinary wave front is greater than extraordinary wave front except along optic axis.

❖ Positive Crystal and Negative Crystal:

Positive Crystal		Negative Crystal	
1.	In positive crystals the refractive index for E-ray is greater than refractive index for O-ray i.e. $\mu_e > \mu_o$.	1.	In negative crystals the refractive index for O-ray is greater than reflective index for e-ray i.e. $\mu_o > \mu_e$
2.	In positive crystals E-ray travels slower than o-ray in all directions except along the optic axis. $V_o > V_e$	2.	In negative crystals O-ray travels slower than E- ray in all directions except along the optic axis i.e. $V_o < V_e$
3.	According to Huygen's ellipse corresponding to e-ray is contained within the sphere corresponding to o-ray	3.	According to Huygen's ellipse corresponding to e-ray lies outside the sphere corresponding to O-ray
4.	Birefringence or amount of double refraction of a crystal is defined as $\Delta\mu = \mu_e - \mu_o$ for $\Delta\mu$ is positive quality for positive crystals	4.	$\Delta\mu$ Negative for negative crystals.
5.	Example: Quartz	5.	Example: calcite

❖ Superposition of waves linearly polarized at right angles.

Let consider two light waves travelling in the x-direction one wave is polarized in x-y plane and the other is polarized y-z plane. Let us find effect produced because of the super positions of these two waves.



Let, these two waves are represented by the following ways;

$$E_y = E_1 \cos(kx - \omega t) \quad (1)$$

$$E_z = E_2 \cos(kx - \omega t + \delta) \quad (2)$$

Where, δ = is phase difference between two waves

$\nu = \omega/2\pi = \text{frequency}$

According to the principle of superposition,

$$E = E_y + E_z = E_1 \cos(kx - \omega t) + E_2 \cos(kx - \omega t + \delta) \quad (3)$$

From equation- (2),

$$E_z = E_2 \cos(kx - \omega t) \cdot \cos \delta - E_2 \sin(kx - \omega t + \delta) \cdot \sin \delta \quad (4)$$

$$= E_2 \cos(kx - \omega t) \cdot \cos \delta \pm [1 - \cos^2(kx - \omega t)]^{1/2} \cdot E_2 \sin \delta$$

From equation- (1),

$$\cos(kx - \omega t) = \frac{E_y}{E_1}$$

$$E_z = E_2 \frac{E_y}{E_1} \cos \delta \pm \sqrt{1 - \frac{E_y^2}{E_1^2}} \cdot E_2 \sin \delta \quad (5)$$

Rearranging above equation,

$$\left[E_z - \frac{E_2}{E_1} E_y \cos \delta \right] = \pm \sqrt{1 - \left(\frac{E_y}{E_1} \right)^2} \cdot E_2 \sin \delta$$

Squaring both the sides,

$$\left[E_z^2 + \left(\frac{E_2}{E_1} E_y \right)^2 \cos^2 \delta \right] - \frac{2E_y E_z E_2}{E_1} \cos \delta = E_2^2 \sin^2 \delta - \left(\frac{E_2}{E_1} E_y \right)^2 \sin^2 \delta$$

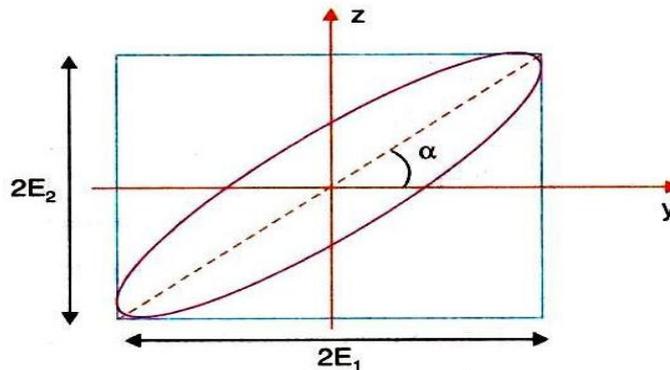
$$E_z^2 + \left(\frac{E_2}{E_1} E_y \right)^2 (\cos^2 \delta + \sin^2 \delta) \cos^2 \delta - \frac{2E_y E_z E_2}{E_1} \cos \delta = E_2^2 \sin^2 \delta$$

$$E_z^2 + \left(\frac{E_2}{E_1} E_y \right)^2 - \frac{2E_y E_z E_2}{E_1} \cos \delta = E_2^2 \sin^2 \delta \quad (6)$$

Dividing both side by E_2^2 ,

$$\left(\frac{E_y}{E_1} \right)^2 + \left(\frac{E_z}{E_2} \right)^2 - \frac{2E_y E_z E_2}{E_1} \cos \delta = \sin^2 \delta \quad (7)$$

Above equation, is the general equation of ellipse, hence, the tip of the resultant vector traces an ellipse in **Y-Z** plane. The ellipse is constrained within a rectangle having sides $2E_1$, and $2E_2$.



➤ **Special cases:-**

1. When $\delta = 0$ or $\pm 2m\pi$, then two waves are in phase.

Therefore,

$$\cos \delta = 1 \quad \text{and} \quad \sin \delta = 0$$

Then, equation (4) becomes,

$$\left(\frac{E_y}{E_1}\right)^2 + \left(\frac{E_z}{E_2}\right)^2 - \frac{2E_y E_z E_2}{E_1} = 0$$

$$\left[\frac{E_y}{E_1} - \frac{E_z}{E_2}\right]^2 = 0$$

$$\frac{E_y}{E_1} - \frac{E_z}{E_2} = 0$$

$$E_z = \frac{E_2 E_y}{E_1} \quad (8)$$

The above equation represents a straight line, having a slope (E_2/E_1) . It means that, the resultant of two plane-polarized waves is again a plane-polarized wave.

2. When $\delta = \pi$ or $\pm (2m\pi + 1)\pi$

The two waves are in opposite phase.

Therefore,

$$\cos \delta = -1 \quad \text{and} \quad \sin \delta = 0$$

Then, equation (4) reduced to,

$$\left(\frac{E_y}{E_1}\right)^2 + \left(\frac{E_z}{E_2}\right)^2 - \frac{2E_y E_z E_2}{E_1} = 0$$

$$\left[\frac{E_y}{E_1} + \frac{E_z}{E_2}\right]^2 = 0$$

$$\frac{E_y}{E_1} + \frac{E_z}{E_2} = 0$$

$$E_z = -\frac{E_2 E_y}{E_1} \quad (9)$$

This equation represents a straight line of a slope $(-E_2/E_1)$. It means that the resultant of two plane polarized waves, which are in opposite phase, is against a plane-polarized wave.

3. If $\delta = \pi/2$ or $\pm (2m\pi + 1)\pi/2$

$$\cos \delta = 0 \quad \text{and} \quad \sin \delta = 1$$

Then, equation (4) reduced to,

$$\frac{E_y}{E_1} + \frac{E_z}{E_2} = 1 \quad (10)$$

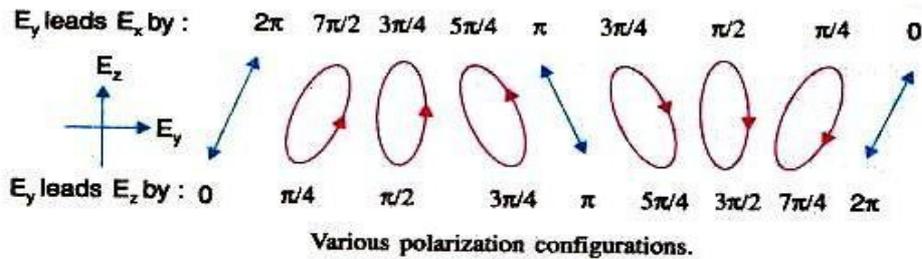
This is the equation of ellipse. Its major and minor axis considers with y-and z coordinates axes. Thus the waves are out of phase by 90° and their resultant wave is elliptically polarized wave.

4. If $\delta = \frac{\pi}{2}$ and $E_1 = E_2 = E_0$

Then, equation (4) reduced to,

$$E_1^2 + E_2^2 = E_0^2$$

This is the equation of circle. Hence result wave is circularly polarized.



❖ TYPES OF POLARIZED LIGHT

1. Unpolarized light:-

It consists of sequence of wave trains, all oriented at random. It is considered as the resultant of two optical vectors. Components, which are incoherent.

2. Linearly polarized light:-

It can be regarded as a resultant of two coherent linearly polarized waves.

3. Partially polarized light:-

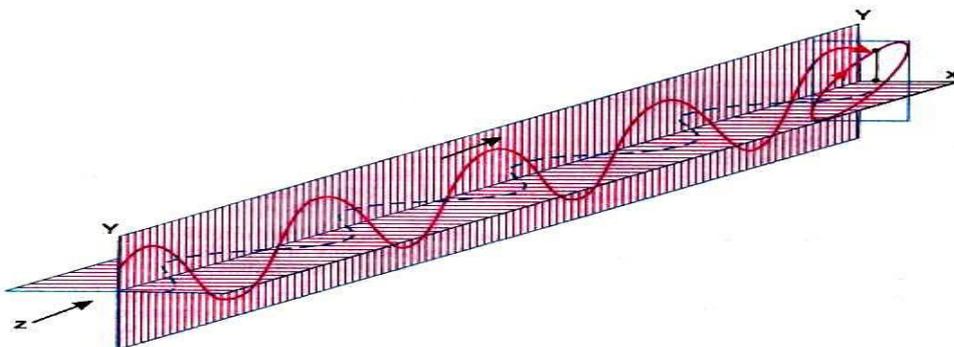
It is a mixture of linearly polarized light and unpolarized light.

4. Elliptically polarized light:-

It is the resultant of two coherent waves having different amplitudes and a constant phase difference of 90° . In elliptically polarized light, the magnitude of electric vector E rotates about the direction of propagation.

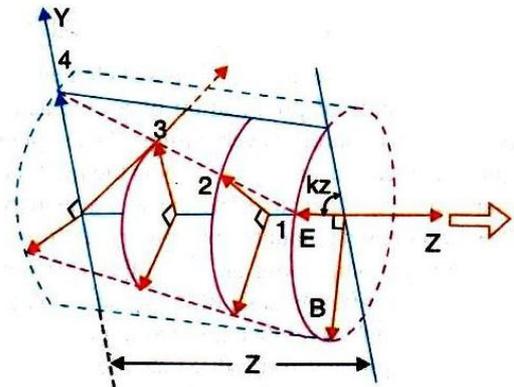
If light is coming towards us, we would observe that tip of the E vector traces an ellipse.

- The side view of E vector gives flattened helix in space.
- If we look from the source and rotation of E vector is clockwise then it is right elliptically polarized wave. If it is anti clock wise then it is left elliptically polarized wave.



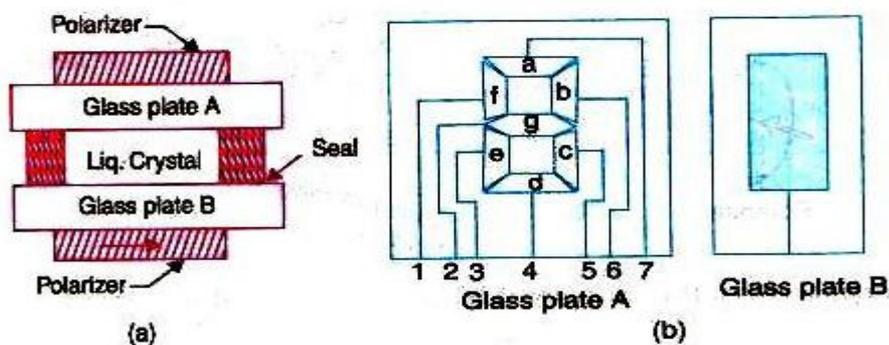
5. Circularly polarized light:-

- It is the resultant of two coherent waves having same amplitudes and a constant phase difference of 90° . In this type of light the magnitude of E vector remains constant.
- If light is coming towards us, we would observe that tip of the E vector traces a circle.
- If we look from the source and rotation E vector is clockwise then it is said right. Circularly polarized and if it anticlockwise then it is left circularly polarized wave.



❖ Liquid Crystal Display (LCD):-

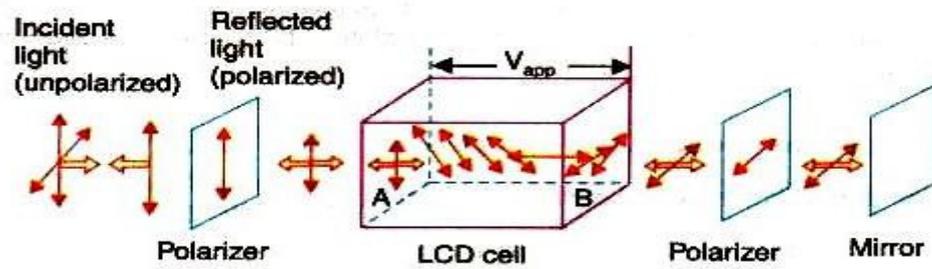
Liquid crystal Display is most widely used device which makes the use of polarization. It is used in wrist watches, educators, clocks, electronic instruments, video games etc.



➤ Construction:-

- An LCD consists of liquid crystal material of $10\mu\text{m}$ thickness.
- It is double refracting material.
- This material is supported between thin glass plates.
- The inner surfaces of thin glass are coated with transparent conducting material.
- This conducting material is etched in the form of a digit or character as shown in fig.2.
- The assembly of glass plates with liquid crystal material is sandwiched between two crossed polarizer sheets.

➤ Working:-



The action of an LCD having a twisted molecular arrangement.

- During fabrication of LCD, the liquid crystal molecules are arranged as shown in fig.3.
- This arrangement of molecules is called twisted molecular arrangement i.e. 90° . Rotation from plate A to B.
- When natural light is incident on the LCD, the front polarizer converts it into linearly polarized light.
- When this polarized light propagates through LCD, the optical vector is rotated by 90° because of twisted molecular arrangement.
- This light passes very easily through the rear polarizer whose transmission axis is perpendicular to that of the front polarizer.
- A reflecting coating at the back of the rear polarizer sends back the light, which comes out from the front polarizer.
- The display seems illuminated uniformly.
- When a voltage is applied to the device, the molecules between the electrodes align along the directions of field.
- When light passes through this region optical vector does not undergo rotation.
- The rear polarizer blocks the light and therefore a dark digit or character is seen in that region as shown in figure.

